Could A Road-centre Line Be An Axial Line In Disguise?

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Abstract

Axial analysis is one of the fundamentals of space syntax. Hillier has proposed that it picks up qualities of configurational relationships between spaces not illuminated by other representations. However, critics have questioned the absolute necessity of axial lines to space syntax, as well as the exact definition of axial lines. An often asked question is: why not another representation? In particular, why not road-centre lines, which are easily available in many countries for use within geographical information systems (GIS)?

The major difference between road-centre line networks and axial networks is that road-centre lines are broken across junctions, and therefore graph measures of the corresponding road-centre networks tend to vary with physical distance rather than the changes of direction as measured within axial networks. As a solution it is has been proposed that we abandon both changes of direction and physical distance as graph measures: instead we should use the angular change between segments. There are strong theoretical and cognitive arguments for this new approach; however, to date, no quantitative studies have been undertaken.

Here we show how angular analysis may be applied generally to road-centre line segments or axial segments, through a simple normalisation procedure that makes values between the two maps comparable. This allows us to make comparative quantitative assessments for a real urban system. As a test case, we take the Barnsbury area of north London, and show that the new algorithm produces good if not better correlation with vehicular flow than standard axial analysis. Furthermore, both our theory and results show that a normalised version of the graph measure of betweenness (or "choice" as it is known within space syntax) is a better model of movement than integration; this further implies there is a strong case for a review of how we use axial analysis in practice.

1. Introduction

Recently there have been many proposals for new variants of space syntax. Most are concerned with axial line analysis, and most, it appears, are trying to shoehorn the analysis into geographical information science (GISci) (e.g., Jiang and Clarumunt, 2002; Batty, 2004; Cutini et al, 2004). These new formulations tend to automate procedures to locate key aspects of skeletal structure to form a new topological representation of urban systems. Whilst some are sympathetic to the epistemic values of space syntax, others are not, and in these cases the purpose of the new representation can be questionable. Within the space syntax community, there has also been a push to retrieve an algorithm for the generation of the axial map itself (Peponis et al, 1998; Turner et al., 2005). However, all these studies have highlighted the fundamental inconsistencies of any representation. At some point, a cartographer must decide whether or not to include a certain feature of the environment or not. The choice will affect whatever resulting topological network is drawn, and thus any representation is unlikely to be reproducible algorithmically. This fallibility, of course, has been appreciated within the space syntax community. Hillier has maintained that the axial representation is "an approximation to the underlying nature of space [as understood through the occupant]" (Hillier, 2003). Indeed, the original rationale for the construction of axial lines and convex spaces was merely to formalise aspects of "stringy" and "beady" systems (Hillier and Hanson, 1984). Ultimately, the implication is that we should not be overly precious about our representation. If we leave our exact representation undefined, then we are lead to the question: is there a representation free method of analysing systems? The answer, of course, must be no, since any deviation in the construction of the accompanying graph will, in general, lead to differences in the values of an analytic result. However, it should be possible to devise a system that ensures an analysis with a minimum of interference from the personal preferences of the cartographer. In this paper, we discuss how such a system can be created for the analysis of topological networks.

This paper begins with a background to angular segment analysis, and examines why it is appropriate as a backbone for a "minimum interference" analysis of topological networks. However, angular segment analysis is not just sensible because it can minimise cartographic differences; there are also strong cognitive grounds for why it should be a good model of pedestrian (and perhaps vehicular) movement. With this in mind, the methodology proposed thrashes out what the most appropriate model of movement should be, and how it should be applied. It is then shown that this model is indeed also the best empirical model of vehicular movement, with a correlation of up to $R^2 = 0.82$ in an application dataset from the Barnsbury area in London. These preliminary results are encouraging, and they lead to a further conclusion: if space syntax is to be used as a model of movement, then we can and should incorporate ideas from traffic modelling to make a fully coherent model of the built environment. By doing so, we may well find that we increase our understanding of how society and space interact.

2. Background

Angular segment analysis (ASA) has recently seen an upsurge in usage within space syntax. In essence, the analysis breaks axial lines into segments, and then records the sum of the angles turned from the starting segment to any other segment within the system (Turner, 2001; more details follow in the section on methodology). This angular sum is treated as the "cost" of a putative journey through the graph, and from it a shortest (that is, least cost) path from one segment to another across the system can be calculated. Most recently, Iida and Hillier (2005) have demonstrated that there is excellent correlation between various ASA measures and movement in four areas of London, including a standard dataset for the Barnsbury area (published in Penn and Dalton, 1994), and it seems that the analysis is set to become one of the mainstream tools of space syntax. In some ways, we should not be surprised at the success of the measures, as there is a good justification for the methodology from within the field of cognitive science. Cognitive scientists have long suggested that angle of turn has much to do with how people perceive the world (Sadalla and Montello, 1989; Montello, 1991). Within space syntax, there is also a precedent for angular analysis, stretching back to Penn and Dalton (1994), who show that "rats" (agents programmed with rules to guide them through the urban maze) which use least-angular strategies for reaching their goals correlate well with patterns of pedestrian movement. More recently, Conroy Dalton (2003) conducts experiments to show how that people tend to minimise angle towards their destination. Furthermore, Turner (2000) proposes a way in which an angular strategy can be taken from the agent-based domain and used within graph theory. Turner, like many before him, proposes a measure of "betweenness", or "choice" as it is called in space syntax. Choice works as follows: for all pairs of possible origin and destination locations, shortest path routes from one to other are constructed. Whenever a node is passed through on a path from origin to destination, its choice value is incremented. Thus frequently used nodes take high values and while those that fall on fewer paths take low values. As many researchers have noted "choice" seems to be a more intuitive model for movement than the traditional measure of integration. However, there is a worry that choice is drawn towards densely packed systems of lines, such as found on housing estates. Herein, we show both how to avoid this potential problem with choice, and how a better correlation can be achieved within the Barnsbury area than that presented by Iida and Hillier.

Recently, the representational problems associated with axial lines have been highlighted by Ratti (2004). Ratti demonstrates that there may be a change of phase where one axial line suddenly becomes many axial lines due to an apparently minor shift in configuration. Whether or not Ratti's example leads to a change of cognitive value is open to question: at one point the single line may be perceived as a single street, whereas the next, where the system may require many lines, it may well be considered a meandering road by the occupant of the system. However, whilst the change makes a difference to axial analysis, it does not cause a problem for ASA, since the split axial lines are only at small angle to each other and the sum weight is approximately equal. Nevertheless, I will argue later that Ratti is right, there are segmented systems where cognitively similar situations result in radically different measurements. We will have to think far more carefully about whether or not any analysis is justifiable according to whether or not it would vary under a different representation.

So what of other representations? Dalton et al (2003) have already demonstrated the possibility that ASA may be applicable to more than axial maps. Previously, it was thought difficult to apply axial analysis measures to road-centre lines, due to the fact that road-centre lines may break across junctions, resulting in the segment problem. The segment problem is that axial lines, when broken into segments, have associated higher transfer "cost" than the straight line, because each step to the next segment incurs a penalty. One response is merely to make the segments continuous, by joining lines that continue in the current direction (Thomson, 2003). Another response is to use angular analysis. In angular analysis, because there is no angular turn to another segment that leads straight on, there is no associated cost, and thus a path that continues in the current direction is by definition continuous across the junction. Dalton et al apply a variant of angular analysis called fractional analysis (Dalton, 2001), to show that both geographic road data (in the form of the US national TIGER lines) and axial data have qualitatively similar patterns of analysis values. This should of course come as no surprise to us: the total amount of turn involved on a route between an origin and destination should be approximately the same, regardless of how the topological skeleton is drawn, be it with road-centre lines or axial lines. However, Dalton et al highlight the relativisation problems between the TIGER data with many lines and the axial data with far fewer lines. In order to fully integrate axial and road-centre lines, we need an analysis free from the effects of numbers of lines, and the answer, I suggest, is to consider the length of segments.

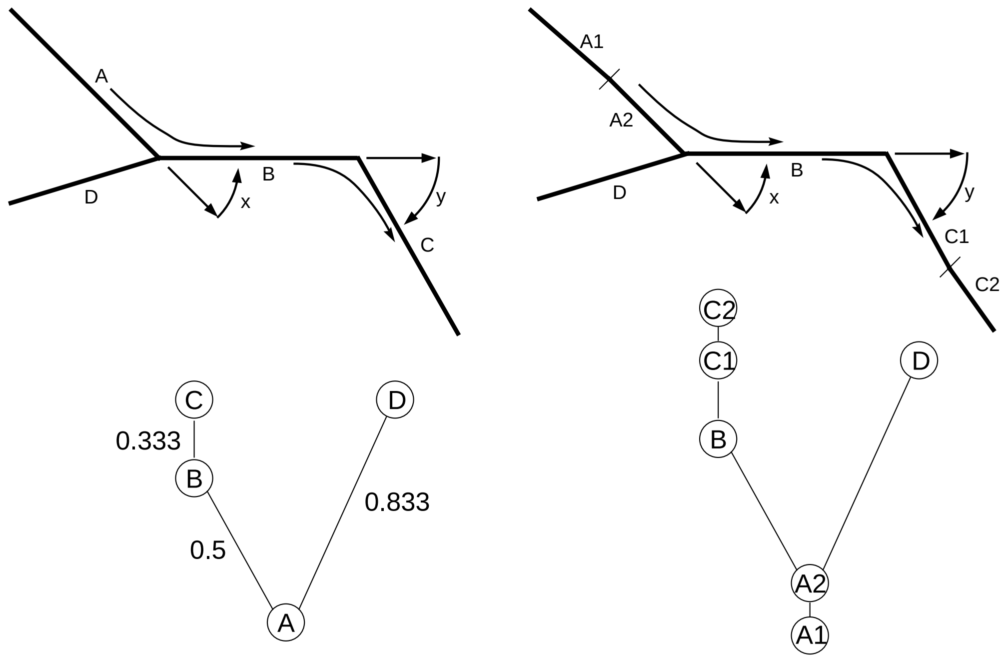

Figure 56: Paths through a network and their associated j-graphs

3. Methodology

We start this section with a brief review of how angular segment analysis (ASA) works, before turning to the angular measures of mean depth and choice. For each measure, we will show how a length-weighted version might be expected to make it representation independent. We then argue that radius should also be length weighted in the same way, that is, it should simply be a physical distance, and discuss of the implications for syntactic measures in general.

3.1. Angular segment analysis

Figure 56 (left) shows a simplified segmented axial map and its associated j-graph from segment. Firstly, notice that we have simply removed the "stubs" that overhang in a standard axial map to form a skeletal network of the system; stub removal can easily be automated, by using a segmentation routine that cuts off any stub of greater than, for example, 25% of the overall length of the line. Later, we will demonstrate that removing the stubs actually makes little difference to the analysis of the system, so here we use it as a simpler conceptualisation.

To calculate the depth to any location, we calculate the total angular turn from one segment to another segment. We follow Iida and Hillier's (2005) convention, and assign a value in the range of 0 (no turn) to 2 (180^o turn) for each turn. So, in figure 1, the depth from segment A to segment B is 0.5 (a turn of 45^o) and the depth to segment C is 0.833 (a turn of 45^o followed by a turn of 30^o - note that the direction of turn is immaterial, the turn angle is always positive). When the system is implemented, it becomes obvious that we are not dealing with a straightforward graph. For example, you cannot move from A to B, then reverse without cost, and move to D, claiming that you have turned 45^o to move to B, and then only 15^o to move from B to D. Directionality is important: you must leave from the segment in the same direction as you arrived. Within our implementation [1]. this is handled by incorporating "back" links and "forward" links for connections at each end of the segment. If you enter via a "back" link, then you must leave via a "forward" link, and vice versa.

3.2. Angular mean depth

We can calculate the mean depth from A using the j-graph as follows: $(B)0.5 + (C)0.833 + (D)0.833/3 = 0.722$. However, what happens when we encounter the situation shown in figure 56 (right)? Here, the cartographer has broken lines A and C to more closely follow the situation on the ground, as is usual when drawing road-centre lines. This leads to a mean depth from A1 of $(A2)0.033 + (B)0.533 + (C1)0.866 + (C2)0.9 + (D)0.866/5 = 0.64$. So this change of representation has made a significant difference to our analysis output. In order to cope with such events, Turner (2001) suggests dividing through by the total angular weight within the system rather than the total number of segments, but the angular weight between one segment and another depends on the direction you are travelling, and so total angular weight varies from segment to segment.

It seems to me to be more intuitive to think about what we are trying to achieve: an analysis, in this case, of movement patterns. We might expect a longer segment to be associated with a higher percentage of origins and destinations of journeys than a shorter segment (at least within an urban area; the same is not true for a motorway). Thus, it seems sensible to relativise systems by weighting our depth measurements by segment length. We can see how this would work to make the two representations in figure 56 equivalent by thinking about segment C. If segment C were 100m long, then its contribution to the mean depth is $100 \times 0.833 = 83.3$. If segment C1 is 70m long, and segment C2 30m long, then their combined contribution is $70 \times 0.866 + 30 \times 0.9 = 87.6$. Obviously, the two are somewhat different due to the extra angles found en route from A1 to C2, but the overall system mean depth is almost unaffected: if we assume C is 100m long as we have done, then the values of weighted mean depth come out as 0.70 and 0.69 for the left and right system respectively.

3.3. Angular choice

Choice, or betweenness as it is called in graph theory, is calculated by generating shortest paths between all segments within the system (i.e., the journey with the lowest angular cost for each possible origin and destination pair of segments). We then sum the flow through each segment according to how many journeys are made through each segment[2].

[1] UCL's Depthmap 5.0 includes all the analysis algorithms described in this paper.

[2] It has been argued that choice is expensive to compute. However, the computational efficiency of a breadth first search to enumerate a shortest path for every OD pair is O(N2) where N is the number segments. Choice increases this to O(N2k), where k is the average path length if we introduce a suitable approximation: if two paths have the same angular cost, then choose one or other of the paths at random, rather than split the flow between them. Where many paths exist, as they do in a segment analysis of all possible OD pairs, this stochastic splitting will lead to a good approximation of the choice measure.

Normally, a value of “1” is assigned to every segment passed through on the shortest path from any origin to any destination. However, as we argued before, longer segments are likely to lead to more journeys, simply because more possible origins and destinations may be fitted along them. Thus, we construct a weighted choice measure by multiplying the length of the origin segment by the length of the destination segment, and this weight is assigned to each segment on the shortest path. The origin and destination themselves are given half this weight, since on average one would start and conclude a journey at the middle of each segment. The mathematical reason for the multiplication is that x origins and y destinations lead to xy paths, so if we keep the sum of lengths Lo for the origins and the sum of lengths L_d for the destinations constant, then no matter the values of x or y, they will always generate a set of paths with total weight L_oL_d. We could of course simply normalise the choice measure, since for any system we know the total number of generated paths for N segments is N^2; however, this normalisation has no effect on the distribution of the measure, which may be distorted by areas of shorter segments and other areas of longer segments.

3.4. Radius

Radius measures are used within space syntax to avoid edge effect or to observe a local phenomenon. Radius suffers similarly to mean depth or choice under different representations, as the number of segments away from a particular location is open to the number of segments that a cartographer uses to represent the feature. As we suggested with mean depth, radius could be set up as an angular cost limit, but this raises another common objection with space syntax: very long lines will allow zero transfer cost along their length. This might be applicable to car journeys, but a pedestrian will surely not generally walk the length, for example, of the Edgware Road. In addition, if we are to avoid edge effect (distortions in values due to where we choose to draw the boundary of the graph) it is difficult to constrain a system where certain nodes may connect much more readily into particular corners of the graph. All this leads to the conclusion that we should use a metric radius for our graphs. This has the beneficial effect that our choice measure covers all journeys within a circle of the defined radius and no more. So, if we know the size of our study area, we can define the exact radius we need to take so as to avoid edge effect.

3.5. Implications for syntactic measures

Ratti (2004) introduces a paradox to space syntax: that a small configurational change can make a major difference to the representation of the system. The paradox also applies to measures of the representation. Figure 57 shows axial representations of a common feature within the Barnsbury area: urban squares. In figure 57(a) the feature is unclear, and one axial line is draw. In figure 57(b) and 57(c), we build up successively wider squares, the single line is gradually split it more lines. There are two problems inherent in this widening. The first is one of relativity. When one line is split into four as in figure 57(b), one gate becomes two gates, and the mean depth increases. However, why should the mean depth increase be commensurate with the flow drop as the occupant takes one path or the other? Indeed, there is also a similar increase in mean depth when we move from figure 57(b) to figure 57(c), but we would surely not expect a similar drop in flow. The second is a problem of flow allocation for the angular choice measure. If the angular turn is higher to the right on figure 57(c) than to the left due to drawing differences. In fact, it could be different

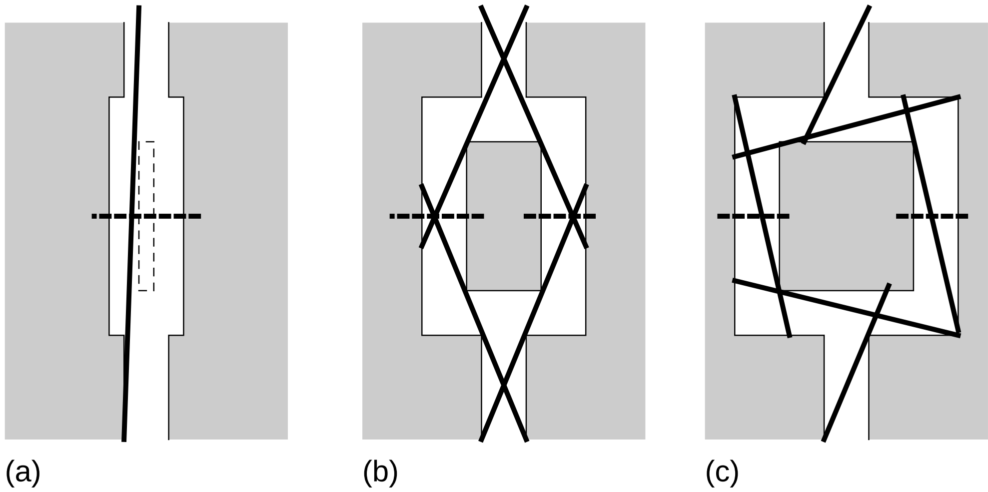

Figure 57: Axial representations of three different urban squares

for the two paths in figure 57(b). However, if there is just a minor deviation, the effect is catastrophic for choice: all the flow will take the shorter route. The traffic modelling community realise this problem, and it has led them to introduce probabilistic models of movement, such as logit or probit models (see, e.g., Bell and Iida, 1997). Both choice and mean depth are affected by problems of representation. However, mean depth has a further problem: it cannot provide a direct model of flow. We typically have a relationship such as $\ln y = m/x + c$, where y is the flow, and x the mean depth. The same is not true of choice: because m units of choice represent y trips, we are able to drop a parameter and write simply $y = mx$.

4. Analysis

As a quantitative test for the representation independent measures, we use data previously gathered for the Barnsbury area in North London, published in Penn and Dalton (1994). Penn and Dalton took gate counts at nearly every possible inter-junction segment within a $1km^2$ area for both vehicular and pedestrian flows. We construct 3km $\times$ 3km axial and road-centre line maps around the study area. We first look at the overall pattern of global measures for axial against road-centre line maps, before moving onto a comparison with all-day average vehicular movement rates. We show that axial measures correlate better than road-centre line measures. However, when metric radius is introduced, the road-centre line model can be improved so that it equals the axial model of movement.

We use Ordnance Survey land-line data to construct the road-centre line map, and an axial map constructed around an original published in Hillier and Hanson (1984). There are a number of problems with using road-centre line data, which are identified by Dalton et al (2003); these include topological links that are missed and seemingly arbitrary decisions about whether or not to include road segments. Furthermore, our road-centre line data is taken 10 years after the initial study. We made minor adjustments to the road

Table 6: Average values for measures of axial and road-centre line maps

	Road-centre line	Axial (with stubs)	Axial (without stubs)
Segment count	20,874	3,933	2,469
Total segment length	163km	204km	150km
Choice	2,023,000	72,039	43,889
Weighted choice	0.90×10^8	1.20×10^8	1.53×10^8
Mean depth	10.77	4.79	4.41
Weighted mean depth	9.90	4.72	4.40

centre lines to ensure that all links were made where two roads met, and to remove links where barriers prevent traffic flow between segments. For the comparison we constructed two axial maps: one with stubs removed where they comprise less than 25% of the line length, and one with the stubs intact. To simplify the graph analysis, we choose to ignore one-way streets.

4.1. Comparison of global measures

Table 6 shows the average values for several measures of the three maps. The road-centre line map has many more segments than either axial map. As might be expected, the total segment length of the road-centre line map is somewhere in between the axial map with stubs and that without. The relativisation of choice is encouraging, as it at least brings the hugely varying choice figures to within the right order of magnitude (figure 58 shows weighted choice for the road-centre line map, figure 59 for the axial map without stubs). However, the mean depth shows that the systems are very differently configured. This is because many more turns are modelled in road-centre lines, as meanderings along streets are included in the representation.

However, overall comparison of the values is of limited use, as the measures might have similar average values but be distributed very differently. In order to compare distributions of values, we use Penn and Dalton's 116 observation gates (figure 60). Note that some gates are on pedestrian paths, so not all can be used for the traffic comparison.

Table in fig 61 shows a comparison of the global measures of the three maps using linear regression. We first normalise the data by dividing through by the highest observed value of each measure or count, and then use a simple function to adjust it to an approximately normal distribution. We apply two different functions: we wish to ensure the choice model is measured as objectively as possible, so the choice and traffic count data is converted to an approximate normal distribution using the cube root of the normalised values. Mean depth does not perform at well with this function, so in order to show the best possible outcome, it was compared with logged values of traffic movement. Table in fig 61 excludes the axial map with stubs, as the correlation between it and the map without stubs is $R^2 > 0.99$ for every pair of variables compared, and the version without stubs seems more appealing from a network perspective.

We first regard the correlation between the measures themselves, before moving on to the correlation with traffic flow. The most obvious and somewhat surprising finding is that weighting the values has little effect on the distribution of the measures. Within each map, there is a strong correspondence between unweighted and weighted measures ($R^2 > 0.95$ in all cases). However, weighting does have some effect if we wish to compare

Figure 58: Weighted choice for a 3km × 3km system of road-centre lines. Thicker lines have higher values.

Figure 59: Weighted choice for a 3km × 3km system of axial segments. Thicker lines have higher values.

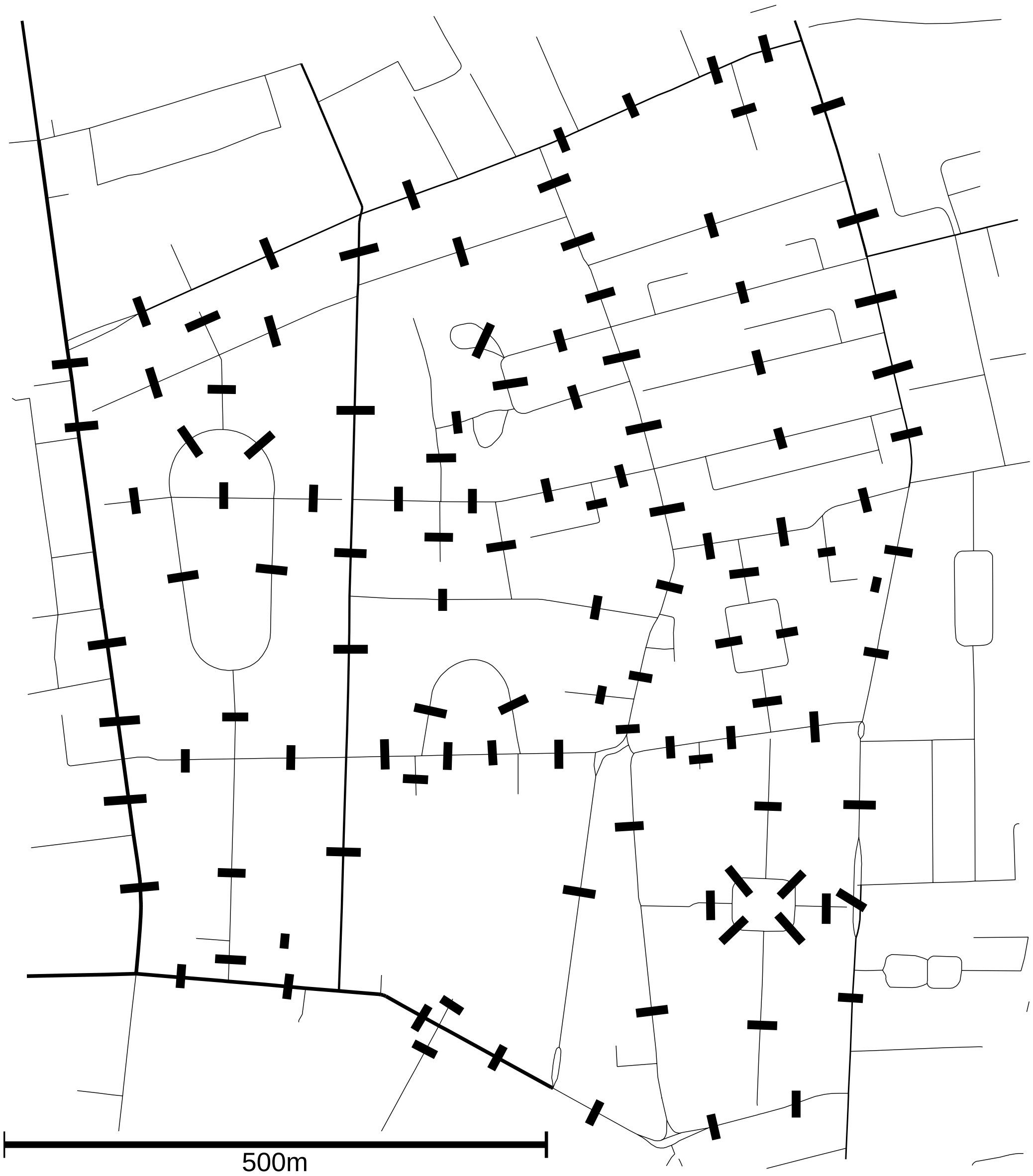

Figure 60: 116 gate locations within the Barnsbury area of North London.

Angular Segment Analysis Correlation (R^2)	Traffic Flow: Modelled	Traffic Flow: Best Fit	Axial (no stubs): Choice	Axial (no stubs): Choice (W)	Axial (no stubs): Mean Depth	Axial (no stubs): M D (W)	Road Centre Line: Choice	Road Centre Line: Choice (W)	Road Centre Line: Mean Depth	Road Centre Line: M D (W)
Axial (no stubs)										
Choice	0.79	0.79	1.00							
Weighted Choice	**0.81**	**0.81**	0.98	1.00						
Mean Depth	n/a	0.63	0.74	0.72	1.00					
Weighted Mean Depth	n/a	0.64	0.74	0.73	1.00	1.00				
Road Centre Line										
Choice	0.70	0.76	0.70	0.76	0.54	0.56	1.00			
Weighted Choice	**0.65**	**0.72**	0.68	**0.75**	0.49	0.52	0.97	1.00		
Mean Depth	n/a	0.50	0.51	0.55	0.65	0.68	0.57	0.57	1.00	
Weighted Mean Depth	n/a	0.51	0.51	0.56	0.65	**0.68**	0.58	0.59	1.00	1.00

Figure 61: Matrix of R^2 correlation coefficients for measures of axial and road-centre line maps.

between maps. Weighting the choice within the axial map brings it further into line with the choice measures of the road-centre line map. Weighting the mean depth has a similar, but less defined, effect.

In both maps the choice method is a considerably better correlate with traffic flow than mean depth. In table in fig 61, the “best-fit” line is simply of the form $y = mx + c$, whereas the “modelled” line has the form $y = mx$, which is inapplicable to mean depth. It is noticeable that both measures are worse for the road-centre line map than they are for the axial map. However, we should not expect too much of the figures at this stage: there is surely significant edge effect in the two systems, with only a 1km border around the study area. The other noticeable feature is that weighting choice for the road-centre line map makes it a considerably worse correlate with traffic flow. At this point, it is tempting to use standard choice rather than weighted choice for our continued experiments with radius measures. However, the unweighted choice measure is a complete artefact of the cartography. In general, curved streets will have more segments than straighter streets, and therefore, within unweighted choice, account for more journeys; that curved streets really will be the origin and destination of more journeys than straight ones of the same length is clearly ludicrous. Therefore, despite the better results found for road-centre line standard choice, we continue with the weighted choice for the following experiments.

4.2. Comparison of radius measures

For radius measures, we will concentrate on the correlation between traffic flow and the modelled weighted choice measure. Radius measures were constructed in the range 1000m to 4000m. Figure 62 shows the general effect of radius for the road-centre line map. At R1000m the lower half of the movement is well modelled, whereas above a certain threshold the model breaks down because longer journeys that are supported by the major thoroughfares are not included in the model. As we increase the radius to 2000m, this effect disappears, and we see the data approach linearity. There are many observations we could make about the groups of outliers observed, but composition of the very similar

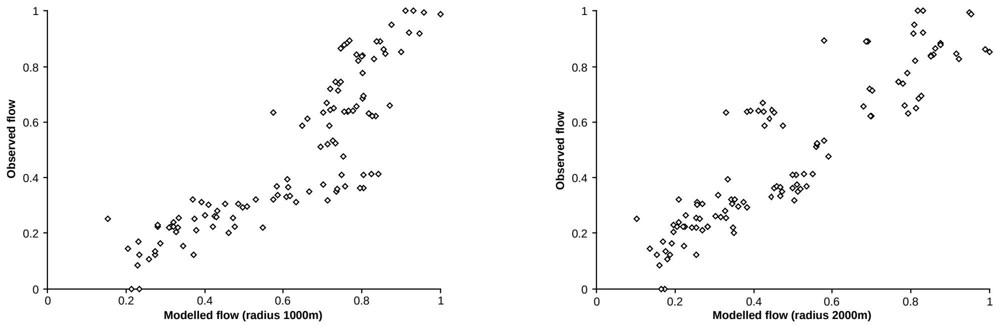

Figure 62: Scatterplots of vehicular movement against weighted choice at radius 1000m and radius 2000m.

groups found in other analyses has already seen significant analysis by Penn and Dalton (1994).

Figure 63 shows how the correlation between observed flow and the weighted choice measure varies with radius for both the axial map and the road-centre line map. The road-centre line map acts as might be expected, with peak correlation at around 2500m, and then tailing off as the edge effect increases. The same is not true for the axial map, which simply approaches its peak correlation as we approach radius n. It appears that two different regimes act for the two different types of systems, although the maximum correlation in both cases is about $R^2 = 0.82$.

5. Conclusion

In this paper we have shown how road-centre line maps and axial line maps may be analysed in a comparable fashion by using angular segment analysis (ASA). We discussed the implementation of segment length-weighted versions of mean depth and betweenness (or choice) in order to make the measures of the two representations equivalent. We have argued that choice should form a better model of movement than mean depth due to the fact that there is an underlying explanatory model, and that the model is directly proportional to the observed value. We then went on to show an analysis of the Barnsbury area of North London. We applied the measures to three 3km x 3km maps of the region: one segmented axial map, one segmented axial map with overhanging "stubs" removed, and one road-centre line map. We found that the axial maps were almost equivalent in terms of correlation of measures, and that weighting the values made only a small difference to the comparison of the maps. We perhaps should not be surprised by this, since the individual maps are drawn with a degree of consistency in terms of line lengths. We correlated the various measures against traffic movement, as observed by Penn and Dalton (1994) at 116 gates within the centre of the study region. We found that choice was a significantly better correlate than mean depth. When we applied the weighted choice measures limited by a metric radius, both axial and road-centre line maps yielded a correlation $R^2 = 0.82$. However, while the road-centre line map seemed limited by edge effect, and peaked in the range radius 2000m to 3000m, the axial map measure monotonically increased to a peak at radius n. It is unclear why this should be the case, and further experimentation with

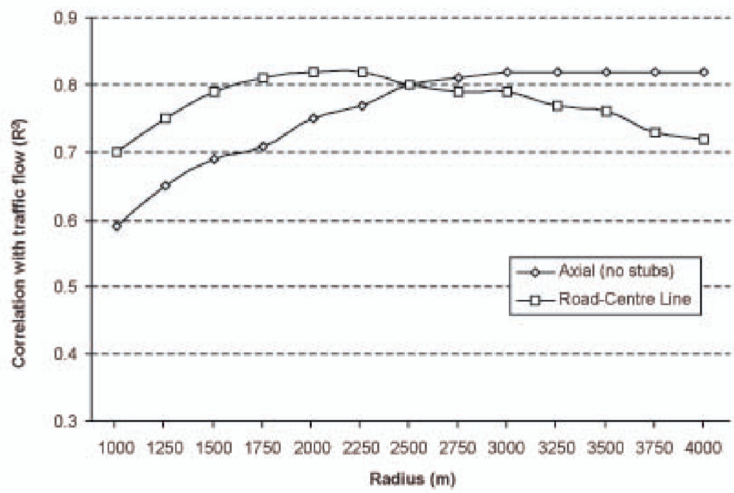

Figure 63: R^2 correlation coefficients for vehicular flow against weighted choice for axial and road-centre line maps.

larger maps is required to see if the two measures reach equivalence.

In conclusion, it seems that the success of metric radius and weighted choice may foster improved syntactic measures. Those measures would still stress the importance of configuration, but would be based on plausible cognitive and physical constraints. For example, we might base a measure on Conroy Dalton's (2003) "British Library" hypothesis, where it is supposed that people minimise the angle to their destination at each decision point, or revisit Penn and Dalton's pedestrian "rats". Indeed, simply because the theory for angular minimisation does come from pedestrian research, it seems natural that the method will achieve success outside traffic analysis. Finally, there is a temptation to view traffic engineering as parameter laden and reminiscent of the large scale models of the '60s; however, cognitive issues are currently breaking through in route choice theory (Nakayama et al, 2001; Cascetta et al, 2002), and we have lessons to learn, especially, how measures of probabilistic route choice such as logit and probit models might be incorporated into space syntax.

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